

SM3 4.5 Exponential and Log Equations

Solve the following equations

1. $3^{n-2} = 27$

$$\begin{aligned}\log_3 3^{n-2} &= \log_3 27 \\ n - 2 &= 3 \\ n &= 5\end{aligned}$$

2. $2^{3x+5} = 128$

$$\begin{aligned}\log_2 2^{3x+5} &= \log_2 128 \\ 3x + 5 &= 7 \\ 3x &= 2 \\ x &= \frac{2}{3}\end{aligned}$$

3. $5^{n-3} = \frac{1}{25}$

$$\begin{aligned}\log_5 5^{n-3} &= \log_5 \frac{1}{25} \\ n - 3 &= -2 \\ n &= 1\end{aligned}$$

4. $10^{x-1} = 100^{2x-3}$

$$\begin{aligned}10^{x-1} &= (10^2)^{2x-3} \\ 10^{x-1} &= 10^{4x-6} \\ \log 10^{x-1} &= \log 10^{4x-6} \\ x - 1 &= 4x - 6 \\ -3x &= -5 \\ x &= \frac{5}{3}\end{aligned}$$

5. $\log_9 x = 2$

$$\begin{aligned}9^{\log_9 x} &= 9^2 \\ x &= 81\end{aligned}$$

6. $\log_{25} n = \frac{3}{2}$

$$\begin{aligned}25^{\log_{25} n} &= 25^{\frac{3}{2}} \\ n &= 5^3 \\ n &= 125\end{aligned}$$

7. $\log_{\frac{1}{7}} x = -1$

$$\begin{aligned}\frac{1}{7}^{\log_{\frac{1}{7}} x} &= \frac{1}{7}^{-1} \\ \frac{1}{7} &= \frac{1}{7} \\ x &= 7\end{aligned}$$

8. $\log(x^2 + 1) = 1$

$$\begin{aligned}10^{\log(x^2+1)} &= 10^1 \\ x^2 + 1 &= 10 \\ x^2 &= 9 \\ x &= \pm 3\end{aligned}$$

9. $\log_b 64 = 3$

$$\begin{aligned}b^{\log_b 64} &= b^3 \\ 64 &= b^3 \\ 4 &= b\end{aligned}$$

10. $\log_5 5^{6n+1} = 13$

$$\begin{aligned}6n + 1 &= 13 \\ 6n &= 12 \\ n &= 2\end{aligned}$$

11. $\log_5 x = \frac{1}{2}$

$$\begin{aligned}5^{\log_5 x} &= 5^{\frac{1}{2}} \\ x &= \sqrt{5}\end{aligned}$$

12. $\log_b 121 = 2$

$$\begin{aligned}b^{\log_b 121} &= b^2 \\ 121 &= b^2 \\ \pm 11 &= b\end{aligned}$$

13. $\log_6(2x - 3) = \log_6(x + 2)$

$$\begin{aligned}6^{\log_6(2x-3)} &= 6^{\log_6(x+2)} \\ 2x - 3 &= x + 2 \\ x &= 5\end{aligned}$$

14. $\log_7(x^2 + 36) = \log_7 100$

$$\begin{aligned}x^2 + 36 &= 100 \\ x^2 &= 64 \\ x &= \pm 8\end{aligned}$$

15. $\log_3 5 + \log_3 x = \log_3 10$

$$\begin{aligned}\log_3 5x &= \log_3 10 \\ 3^{\log_3 5x} &= 3^{\log_3 10} \\ 5x &= 10 \\ x &= 2\end{aligned}$$

16. $\log_4 a + \log_4 9 = \log_4 27$

$$\begin{aligned}\log_4 9a &= \log_4 27 \\ 4^{\log_4 9a} &= 4^{\log_4 27} \\ 9a &= 27 \\ a &= 3\end{aligned}$$

$$17. \log 16 - \log 2t = \log 2$$

$$\begin{aligned} \log \frac{8}{t} &= \log 2 \\ 10^{\log \frac{8}{t}} &= 10^{\log 2} \\ \frac{8}{t} &= 2 \\ \frac{t}{8} &= \frac{1}{2} \\ t &= 4 \end{aligned}$$

$$18. \log_7 24 - \log_7(y+5) = \log_7 8$$

$$\begin{aligned} \log_7 \frac{24}{y+5} &= \log_7 8 \\ 7^{\log_7 \frac{24}{y+5}} &= 7^{\log_7 8} \\ \frac{24}{y+5} &= 8 \\ \frac{y+5}{24} &= \frac{1}{8} \\ y+5 &= 3 \\ y &= -2 \end{aligned}$$

$$19. \log_2 n = \frac{1}{4} \log_2 16 + \frac{1}{2} \log_2 49$$

$$\begin{aligned} \log_2 n &= \log_2 16^{\frac{1}{4}} + \log_2 49^{\frac{1}{2}} \\ \log_2 n &= \log_2 2 + \log_2 7 \\ \log_2 n &= \log_2 14 \\ 2^{\log_2 n} &= 2^{\log_2 14} \\ n &= 14 \end{aligned}$$

$$20. 2 \log 6 - \frac{1}{3} \log 27 = \log x$$

$$\begin{aligned} \log 6^2 - \log 27^{\frac{1}{3}} &= \log x \\ \log 36 - \log 3 &= \log x \\ \log 12 &= \log x \\ 10^{\log 12} &= 10^{\log x} \\ 12 &= x \end{aligned}$$

$$21. \log z + \log(z+3) = 1$$

$$\begin{aligned} \log(z^2 + 3z) &= 1 \\ 10^{\log(z^2 + 3z)} &= 10^1 \\ z^2 + 3z &= 10 \\ z^2 + 3z - 10 &= 0 \\ (z+5)(z-2) &= 0 \\ z &= \{-5, 2\} \end{aligned}$$

As -5 fails to check, we eliminate it.

$$z = 2$$

$$22. \log_6(a^2 + 2) + \log_6 2 = 2$$

$$\begin{aligned} \log_6(2a^2 + 4) &= 2 \\ 6^{\log_6(2a^2 + 4)} &= 6^2 \\ 2a^2 + 4 &= 36 \\ 2a^2 &= 32 \\ a^2 &= 16 \\ a &= \pm 4 \end{aligned}$$

$$23. \log_2(12b - 21) - \log_2(b^2 - 3) = 2$$

$$\begin{aligned} \log_2 \left(\frac{12b - 21}{b^2 - 3} \right) &= 2 \\ 2^{\log_2 \left(\frac{12b - 21}{b^2 - 3} \right)} &= 2^2 \\ \frac{12b - 21}{b^2 - 3} &= 4 \\ 12b - 21 &= 4b^2 - 12 \\ 4b^2 - 12b + 9 &= 0 \\ (2b - 3)^2 &= 0 \\ 2b - 3 &= 0 \\ 2b &= 3 \\ b &= \frac{3}{2} \end{aligned}$$

As $\frac{3}{2}$ fails to check, we eliminate it.

$$b = \emptyset$$

$$24. \log_2(y+2) - \log_2(y-2) = 1$$

$$\begin{aligned} \log_2 \left(\frac{y+2}{y-2} \right) &= 1 \\ 2^{\log_2 \left(\frac{y+2}{y-2} \right)} &= 2^1 \\ \frac{y+2}{y-2} &= 2 \\ y+2 &= 2y-4 \\ -y &= -6 \\ y &= 6 \end{aligned}$$

25. $\log_3 0.1 + 2 \log_3 x = \log_3 2 + \log_3 5$
 $\log_3 0.1 + \log_3 x^2 = \log_3 2 + \log_3 5$
 $\log_3 0.1x^2 = \log_3 10$
 $3^{\log_3 0.1x^2} = 3^{\log_3 10}$
 $0.1x^2 = 10$
 $x^2 = 100$
 $x = \pm 10$
 As -10 fails to check, we reject it.
 $x = 10$

26. $\log_5 64 - \log_5 \frac{8}{3} + \log_5 2 = \log_5 4p$
 $\log_5 128 - \log_5 \frac{8}{3} = \log_5 4p$
 $\log_5 \frac{128}{8/3} = \log_5 4p$
 $\log_5 128 \cdot \frac{3}{8} = \log_5 4p$
 $\log_5 16 \cdot 3 = \log_5 4p$
 $\log_5 48 = \log_5 4p$
 $5^{\log_5 48} = 5^{\log_5 4p}$
 $48 = 4p$
 $12 = p$

27. $2e^x - 1 = 0$
 $2e^x = 1$
 $e^x = \frac{1}{2}$
 $x = \ln\left(\frac{1}{2}\right)$

28. $-3e^{4x} + 11 = 2$
 $-3e^{4x} = -9$
 $e^{4x} = 3$
 $4x = \ln 3$
 $x = \frac{\ln 3}{4}$

29. $\ln 2x = 4$
 $2x = e^4$
 $x = \frac{e^4}{2}$

30. $\ln 3x = 5$
 $3x = e^5$
 $x = \frac{e^5}{3}$

31. $\ln(x + 1) = 1$
 $x + 1 = e$
 $x = e - 1$

32. $\ln(x - 7) = 2$
 $x - 7 = e^2$
 $x = e^2 + 7$

33. $\ln x + \ln 3x = 12$
 $\ln 3x^2 = 12$
 $3x^2 = e^{12}$
 $x^2 = \frac{e^{12}}{3}$
 $x = \pm \frac{e^6}{\sqrt{3}} = \pm \frac{e^6\sqrt{3}}{3}$
 The negative result fails to check.
 $x = \frac{e^6\sqrt{3}}{3}$

34. $\ln 4x + \ln x = 9$
 $\ln 4x^2 = 9$
 $4x^2 = e^9$
 $x^2 = \frac{e^9}{4}$
 $x = \pm \frac{e^{4.5}}{2}$
 The negative result fails to check.
 $x = \frac{e^{4.5}}{2}$

35. $\ln(x^2 + 12) = \ln x + \ln 8$
 $\ln(x^2 + 12) = \ln 8x$
 $x^2 + 12 = 8x$
 $x^2 - 8x + 12 = 0$
 $(x - 6)(x - 2) = 0$
 $x = \{2, 6\}$

36. $\ln x + \ln(x + 4) = \ln 5$
 $\ln(x^2 + 4x) = \ln 5$
 $x^2 + 4x = 5$
 $x^2 + 4x - 5 = 0$
 $(x + 5)(x - 1) = 0$
 $x = \{-5, 1\}$
 -5 fails to check.
 $x = 1$

$$37. \quad e^{\ln x} = 4$$
$$x = 4$$

$$38. \quad 200e^{-4x} = 15$$
$$e^{-4x} = \frac{3}{40}$$
$$-4x = \ln\left(\frac{3}{40}\right)$$
$$x = -\frac{1}{4}\ln\left(\frac{3}{40}\right)$$

$$39. \quad \ln x^2 = 10$$
$$x^2 = e^{10}$$
$$x = \pm e^5$$

$$40. \quad 9 - 2e^x = 7$$
$$-2e^x = -2$$
$$e^x = 1$$
$$x = 0$$

$$41. \quad \ln \sqrt{x+2} = 1$$
$$\sqrt{x+2} = e$$
$$x+2 = e^2$$
$$x = e^2 - 2$$

$$42. \quad \ln(x-2)^2 = 12$$
$$(x-2)^2 = e^{12}$$
$$x-2 = \pm e^6$$
$$x = 2 \pm e^6$$